

# Verifying Components While Guaranteeing Compositionality

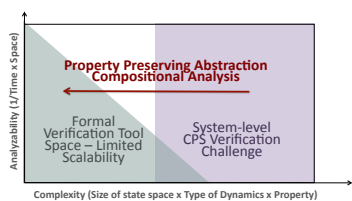
Abstraction-Based Tool and Method Tackles Complexity of CPS Verification

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## CPS Verification: Need and Challenge

- Simulation of CPS with complex dynamics can yield misleading results and does not provide proofs
- Verification tools cannot handle differential equations of CPS dynamics

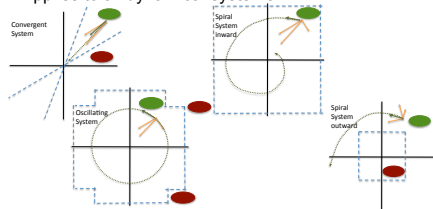
## Solution: Abstraction and Compositionality



Qualitative Abstraction	Relational Abstraction
<ul style="list-style-type: none"> <li>Abstracts state space</li> <li>E.g., dynamics abstracted to increasing/decreasing or positive/negative</li> </ul>	<ul style="list-style-type: none"> <li>Abstracts transition relation</li> <li>E.g., dynamics abstracted to increasing/decreasing at certain rate or amount of pos/neg</li> </ul>

## Relational Abstraction

- Applies to all dynamical systems

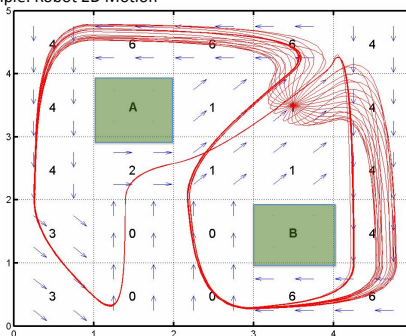


- Effective relational abstractions can be automatically computed for several system dynamics classes

Class	$\frac{d\tilde{x}}{dt}$	Relational Abstraction
Timed Systems	$\dot{\tilde{x}} = 1, \dot{\tilde{y}} = 1$	$\tilde{x}' - \tilde{x} = \tilde{y}' - \tilde{y}$
Multirate Systems	$\dot{\tilde{x}} = 2, \dot{\tilde{y}} = 3$	$\frac{\tilde{x}' - \tilde{x}}{2} = \frac{\tilde{y}' - \tilde{y}}{3}$
Linear Hybrid Systems	$\dot{\tilde{x}} = A\tilde{x}$	$0 \leq p' \leq p \vee 0 \geq p' \geq p, p = \vec{c}^T \tilde{x}, \vec{c}$ Eigenvector of $A^T$ corr. to neg. eigenval

## Demonstrated Verification on Several Hybrid System Benchmarks

Example: Robot 2D Motion



- Does robot stay clear of A and B?

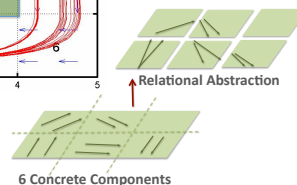
- System dynamics:

$$\dot{\tilde{x}} = \tilde{v}$$

$$\tilde{v} = A(\tilde{v} - \tilde{v}_d)$$

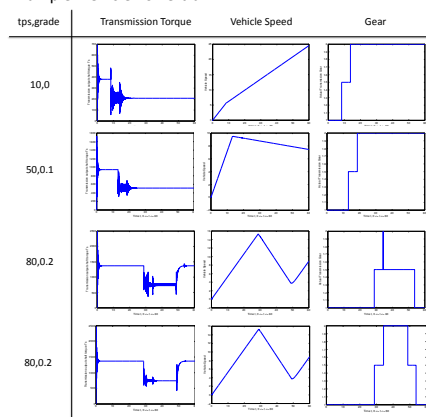
- The direction  $\tilde{v}_d$  depends on the position in the grid.

- Rel. Abstraction tool verifies instances in minutes
- From [Ansgar and Ivancic, 2004]



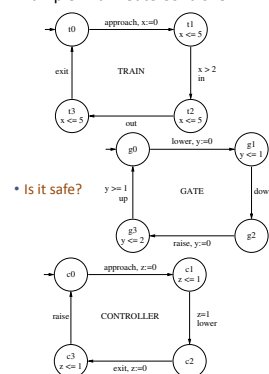
**Rel. Abs. done for each mode/component, each corresponding to open system**  
**Verification results hold for composed system**

Example: Vehicle Powertrain



- Does there exist "second-to-first-to-second" gear transition?

Example: Train Gate Controller

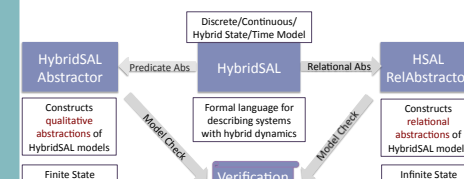


From [Dutertre and Sorea, 2004]

## Approach

- Relational Abstraction is an over-approximation of the transitive closure of the transition relation
- Useful for proving safety properties and establishing conservative safety bounds

## Automated Verification Tool



The results produced by abstraction techniques enable **compositional verification** when components are put together to build systems

Other Work: Few automated tools for verifying systems with mixed discrete/continuous dynamics, and none are compositional

## Benefits

- Enables analyzability of complex systems
- On Hybrid System benchmarks, verification time reduces from 10 hours to few minutes (**100x improvement**)

## Feature

- Compositional analysis handles open components with hybrid dynamics

## Best of Breed

- Approach compatible with other abstraction and model-checking techniques

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